FRICTION AND HEAT TRANSFER IN TURBULENT GAS FLOW BEHIND AN ACCELERATING PISTON

A. M. Bubenchikov and S. N. Kharlamov

Unsteady turbulent gas motion and heat transfer are examined on a circular cylindrical tube segment bounded at one end by a fixed wall and at the other by a piston capable of displacement. At the initial instant the pressure, density, and temperature of the gases in the domain between the wall and the piston substantially exceed the state parameters in the free part of the channel. The tube wall is heat conducting. Under the action of the pressure of compressed gases the piston, of finte mass, acquires significant acceleration and moves to the free end of the tube. Such a formulation is known as the classical Lagrange problem [1] in the scheme to determine the stream characteristics and the piston motion velocity averaged over the channel section.

The purpose of this paper is to investigate the turbulent flow configuration under the conditions being examined and to determine the parameters of the dynamical and thermal action of the stream on the channel wall.

We assume that axial flow symmetry exists. The gas is perfect, axial heat and momentum transfer by diffusion is absent. In this case the gas flow is described by the Reynolds equation in the "narrow channel" approximation [2], which in combination with the energy equations for the gas and the heat conduction equation for the wall have the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \left(\rho u_x\right)}{\partial x} + \frac{1}{r} \frac{\partial \left(\rho u_r r\right)}{\partial r} = 0; \tag{1}$$

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_r\frac{\partial u_x}{\partial r}\right) = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{\Sigma}\frac{\partial u_x}{\partial r}\right), \quad \frac{\partial p}{\partial r} = 0;$$
(2)

$$\rho c_p \left(\frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_r \frac{\partial T}{\partial r} \right) = \frac{dp}{dt} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda_\Sigma \frac{\partial T}{\partial r} \right) + \mu_\Sigma \left(\frac{\partial u_x}{\partial r} \right)^2,$$

$$p = \rho R_{\mathbf{g}} T;$$
(3)

$$\rho_{W}c_{W}\frac{\partial T_{W}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{W}\frac{\partial T_{W}}{\partial r}\right), \quad \frac{dp}{dt} = \frac{\partial p}{\partial t} + \frac{u_{x}\partial p}{\partial x}, \quad \mu_{\Sigma} = \mu + \mu_{t}, \quad \lambda_{\Sigma} = \lambda + \lambda_{t}.$$
(4)

Here, t, x, r are the time and independent space coordinates; u_x , u_r are velocity vector components; ρ , T, p are the density, temperature, and pressure of the gas; μ , λ are the molecular viscosity and heat conduction coefficients; μ_t , λ_t are the turbulent analogs of the transport coefficients; c_p and R_g are the isobaric specific heat and the gas constant; ρ_w , c_w , λ_w are the density, specific heat, and heat conduction coefficient of the wall material; and T_w is the wall temperature. All the quantities in the system (1)-(3) are averaged (average of the turbulent fluctuations). The temperature dependence of the gas viscosity was determined by the Sutherland law [3].

Because the pressure is independent of the radial coordinate, it is expedient to find the distribution p = p(x, t) from the solution of the equations (1)-(3) averaged across the channel. Analogously to [4] for obtaining the pressure we have the following system of one-dimensional nonstationary gasdynamics equations

$$\frac{\partial \left(\bar{\rho}s\right)}{\partial t} + \frac{\partial \left(\bar{\rho}us\beta_{1}\right)}{\partial x} = 0; \tag{5}$$

$$\frac{\partial \left(\bar{\rho}^{u}s\beta_{1}\right)}{\partial t} + \frac{\partial \left(\bar{\rho}^{u}s\beta_{2}\right)}{\partial x} = -s\frac{\partial p}{\partial x} - \chi\tau_{w}; \tag{6}$$

$$\frac{\partial}{\partial t} \left[\left(\frac{p}{(\gamma - 1)} + \bar{\rho} \, \frac{u^2}{2} \, \beta_2 \right) \right] + \frac{\partial}{\partial x} \left[\left(\frac{p}{(\gamma - 1)} + \bar{\rho} \, \frac{u^2}{2} \, \beta_3 \right) us \right] = -\frac{\partial}{\partial x} \left(pus \right) - \chi q_x, \tag{7}$$

where s is the cross-sectional area, χ is its perimeter, and γ is the adiabatic index,

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$$\tau_w = -\mu \frac{\partial u_x}{\partial r}\Big|_{r=R}, \quad q_w = \lambda \frac{\partial T}{\partial r}\Big|_{r=R}, \quad \bar{\rho} = \frac{1}{s} \iint_s \rho \, ds; \tag{8}$$

$$u = \frac{1}{s} \iint_{s} u_{x} ds, \quad \beta_{i} = \frac{1}{\bar{\rho}u^{i}s} \iint_{s} \rho u_{x}^{i} ds \quad (i = \overline{1, 3})$$
(9)

are coefficients of stream inhomogeneity or dispersion of the gasdynamic parameters relative to the values of these quantities averaged over the channel section.

Equations (5)-(7) are an exact corollary of the system (1)-(3). Consequently, no additional assumptions simplifying the mathematical model of the process are utilized for determination of the pressure. Therefore, the numerical solution of the problem can be obtained for each time step in two stages. Equations (5)-(7) are solved in the first, and the pressure distribution and all the averaged flow characteristics are found in the new time layer. Here τ_W , q_W , β_i are taken from the past time layer. Equations (1)-(4) are integrated in the second stage, i.e., two-dimensional fields of the desired quantities are computed and according to (8) and (9) the τ_W , q_W , β_i are determined.

Let us note that in the case of insignificant friction and heat elimination losses and for weak influence of the coefficients β_i , exact analytic solutions of the gasdynamics equations can be applied (see [5], say).

The characteristic flow velocities are quite large in problems of the class under consideration. The range of Reynolds number variation is determined by the interval from 0 to 10^7 , i.e., the near-wall zone is characterized by the presence of large gradients of the desired quantities. Moreover, the domain of variation of the space variables is not stationary. Under these conditions it is expedient to make the change of variables

$$\{t, x, r\} \rightarrow \{t, \xi, \eta\}: t_1 = t, \quad \xi = \int_{x \in \mathbf{t}}^{\infty} \overline{\rho} s \, ds, \quad \eta = \ln\left(1 - \frac{r}{R} + \Delta\right). \tag{10}$$

in the equations governing the processes in the gas phase. Here x_{st} is the Euler coordinate of the left boundary of the flow domain, R is the channel inner radius, Δ is a transformation parameter assuring incidence of the requisite number of points of the difference mesh in the viscous sublayer. Utilization of the substitution (10) permits a stationary domain of integration to be obtained in the plane of the new space coordinates. Let us note that for R = const the transformation (10) is orthogonal. A domain of large temperature gradients exists at the channel wall, as well as in the gas, at the phase interfacial boundary, and therefore, condensation of the difference mesh nodes is necessary even in this part of the computational domain. Moreover, the adjoint formulation of the thermal part of the problem (the heat propagation equations in the gas and wall with boundary conditions of the fourth kind on the phase interfacial boundary) requires the application of matched meshes in the solid and gas phases. All this can be achieved if a change of variables of the kind (10) is made in (4), where $\eta_1 = \ln (R/r - 1 + \Delta_1) (\Delta_1$ is a parameter assuring condensation of the different mesh nodes at the channel inner surface) is taken as the new independent transverse coordinate.

The system of equations transformed in this manner has the form

$$\frac{\partial \rho}{\partial t_1} + w\xi_x \frac{\partial \rho}{\partial \xi} + \rho\xi_x \frac{\partial u_x}{\partial \xi} + \frac{\eta_r}{r} \frac{\partial}{\partial \eta} (\rho u_r r) = 0; \qquad (11)$$

$$\rho\left(\frac{\partial u_x}{\partial t_1} + w\xi_x\frac{\partial u_x}{\partial \xi} + u_r\eta_r\frac{\partial u_x}{\partial \eta}\right) = -\xi_x\frac{\partial p}{\partial \xi} - \frac{\eta_r}{r}\frac{\partial}{\partial \eta}\left(r\mu_{\Sigma}\eta_r\frac{\partial u_x}{\partial \eta}\right), \quad \frac{\partial p}{\partial \eta} = 0; \quad (12)$$

$$\rho c_p \left(\frac{\partial T}{\partial t_1} + w \xi_x \frac{\partial T}{\partial \xi} + u_r \eta_r \frac{\partial T}{\partial \eta} \right) = \frac{\eta_r \partial}{r \partial \eta} \left(r \lambda_\Sigma \eta_r \frac{\partial T}{\partial \eta} \right) + \frac{dp}{dt_1} + \mu_\Sigma \eta_r^2 \left(\frac{\partial u_x}{\partial \eta} \right)^2; \tag{13}$$

$$\rho_{\mathbf{W}}c_{\mathbf{W}}\left(\frac{\partial T_{\mathbf{W}}}{\partial t_{1}}-\xi_{\mathbf{x}}u\frac{\partial T_{\mathbf{W}}}{\partial\xi}\right)=\frac{\eta_{1r}}{r}\frac{\partial}{\partial\eta_{1}}\left(r\lambda_{\mathbf{W}}\eta_{1r}\frac{\partial T_{\mathbf{W}}}{\partial\eta_{1}}\right);\tag{14}$$

$$\frac{\partial v}{\partial t_1} = \frac{\partial us}{\partial \xi}; \tag{15}$$

$$\frac{\partial u}{\partial t_1} = -s \frac{\partial p}{\partial \xi} - \frac{\chi}{s} v \tau_w; \tag{16}$$

$$\frac{\partial e}{\partial t_1} + p \frac{\partial v}{\partial t_1} = \frac{\gamma}{s} v (q_w - u\tau_w), \quad e = \frac{pv}{(\gamma - 1)}.$$
(17)

Here

$$w = (u_x - u); \quad \xi_x = \overline{\rho}s; \quad \eta_r = \frac{1}{R(1 + \Delta) - r}; \quad \eta_{1r} = \frac{1}{r - R(1 - \Delta_1)};$$
$$\frac{dp}{dt_1} = \frac{\partial p}{\partial t_1} + w\xi_x \frac{\partial p}{\partial \xi}; \quad v = \frac{1}{\overline{\rho}}; \quad \beta_i = 1 \quad (i = \overline{1, 3});$$

 τ_w , q_w , $\bar{\rho}$, u are determined from the relationships (8) and (9).

Taking account of the nonstationarity of the change in the turbulent stream configuration was performed on the basis of a model of energy-scale type. The balance equation for the turbulence kinetic energy selected according to [6] is supplemented by a transport differential equation for the integral scale of turbulent fluctuations [7]. In the case of nonstationary compressible gas flow the model is represented in the form

$$\frac{\partial E}{\partial t_{1}} + w\xi_{x}\frac{\partial E}{\partial\xi} + u_{r}\eta_{r}\frac{\partial E}{\partial\eta} = \frac{\eta_{r}}{r}\frac{\partial}{\partial\eta}\left[r\left(\nu + \nu_{t}\right)\eta_{r}\frac{\partial E}{\partial\eta}\right] + \nu_{t}\eta_{r}^{2}\left(\frac{\partial u_{x}}{\partial\eta}\right)^{2} - \frac{b_{2}\left(\nu + b_{1}\nu_{t}\right)}{L^{2}}E;$$

$$\frac{\partial L}{\partial t_{1}} + w\xi_{x}\frac{\partial L}{\partial\xi} + u_{r}\eta_{r}\frac{\partial L}{\partial\eta} = \frac{\eta_{r}}{r}\frac{\partial}{\partial\eta}\left[r\left(\nu + b_{3}\nu_{t}\right)\eta_{r}\frac{\partial L}{\partial\eta}\right] -$$
(18)

$$-b_4 \frac{L}{E} v_i \eta_r^2 \left(\frac{\partial u_x}{\partial \eta}\right)^2 + B b_5 \sqrt{E} \left(1 - \frac{L^2}{\left(R-r\right)^2}\right), \qquad (19)$$

where $b_1 = 0.4$, $b_2 = 3.93$, $b_3 = 0.35$, $b_4 = 0.125$, $b_5 = 0.29$, $B = b_6 + b_7/Re_t$, $b_6 = 0.3$, $b_7 = 1.75$. The relationships [6, 8]

$$v_t / v = \alpha \operatorname{Re}_t \left[1 - \exp\left(-\sigma_2 \operatorname{Re}_t^2\right) + \sigma_3 \operatorname{Re}_t^{1/2} \exp\left(-\sigma_1 \operatorname{Re}_t^2\right) \right],$$
(20)

$$\operatorname{Re}_t = \frac{\sqrt{\varphi E} L}{v}, \quad \alpha = 0.2, \quad \sigma_1 = 4 \cdot 10^{-4}, \quad \sigma_2 = 2.1 \cdot 10^{-4}, \quad \sigma_3 = 2 \cdot 10^{-2},$$

$$\operatorname{Pr}_t = \frac{v_t}{v} \left\{ \left[(2\varkappa_1 \operatorname{Pr})^{-2} + \frac{v_t}{\varkappa_3 v} \left(\frac{v_t}{v} + \frac{1}{\varkappa_2} \right) \right]^{1/2} - (2\varkappa_1 \operatorname{Pr})^{-1} \right\}^{-1},$$

$$\lambda_t = \frac{\mu_t c_p}{\operatorname{Pr}_t}, \quad \varkappa_1 = 0.5, \quad \varkappa_2 = 0.15, \quad \varkappa_3 = 0.9$$

are used to find the coefficients of turbulent momentum and heat transfer. Here φE is the fraction of turbulent mole energy responsible for the exchange processes. The value of φ is defined as follows.

$$\varphi = 1 - \exp\left(-\frac{a}{K_0} \frac{t_1}{t_t}\right),\tag{21}$$

where a = const, $t_t = L/\sqrt{E}$ is the turbulent fluctuation time computed from local values of the turbulence characteristics, $K_0 = ((D/u^2)\partial u/\partial t)_0$ is the nonstationarity parameter, and D = 2R is the channel diameter; the values of quantities characteristic for this process are marked with the subscript 0. In the case of a gas flow behind an accelerating piston we have $u_0 = c_0$, $(\partial u/\partial t)_0 = c_0^2/k_c$, then $K_0 = D/k_c$ (k_c is the chamber length, c_0 is the speed of sound in the gas initially at rest). A relationship analogous to (21) was used earlier to describe turbulent heat and mass transfer in a combustion wave [9]. In this paper (21) is used to investigate accelerated streams. It reflects the exchange process mechanism asymptotically correctly in such flows. As follows from (21), as $t_1 \rightarrow \infty$ the turbulent exchange processes proceed in the same manner as in stationary streams. For $K_0 \rightarrow 0$ and any fixed arbitrarily small t_1 , the effect of nonstationarity is negligibly small. For $K_0 \rightarrow \infty$ in a bounded time interval the heat and momentum transfer of turbulence becomes secondary.

The system (11)-(20) describing gas flow and heat transfer from the tube wall was integrated under the following boundary conditions.

Initial distributions:

$$t_{1} = 0, v = v_{i}, u_{x} = 0, u_{r} = 0, T = T_{i}, E = E_{i}, L = L_{i},$$

$$T_{w} = T_{iw}.$$
 (22)

All the quantities with subscripts are constants characterizing the initial state of the gas and the wall.

Boundary conditions:

$$x = x_{p}(t_{1}), v = V(t_{1}), u_{x} = U(t_{1}), T = \Theta(t_{1}), E = L = 0.$$
(23)

It is assumed in the computations that the left and right boundaries of the flow domain exert no substantial thermal effect on the stream; consequently, the appropriate values of the specific volume, velocity, and temperature of the gas on the piston (23) can be obtained from the solution of the gasdynamic equations (15)-(17).

Symmetry conditions are given on the channel axis

$$r = 0, \, \partial u_x / \partial r = \partial T / \partial r = \partial E / \partial r = \partial L / \partial r = 0.$$
(24)

Conditions of adhesion, and no turbulent fluctuations for the field of gasdynamic quantities adjoint for the temperature are posed on the wall

$$r = R, u_x = u_r = E = L = 0, T = T_{u_r}, \lambda \partial T / \partial r = \lambda \partial T_{u_r} / \partial r.$$
⁽²⁵⁾

The heat conduction equation (14) for which the heat insulation condition

$$r = R_1, \, \partial T / \partial r = 0 \tag{26}$$

is used on the tube outer boundary is solved on the channel wall (R_1 is the channel outer radius).

Conditions of nonpenetration on the flow domain boundaries are boundary conditions for the characteristics, determined from (15), (17), averaged over the channel section.

The results represented in this paper are obtained for helium flows in a steel tube. Computations were executed for the following values of the original parameters: $\ell_c = 0.3 \text{ m}$, $R = 3.95 \times 10^{-3} \text{ m}$, $\sigma = 0.918$, $m = 10^{-3} \text{ kg}$, $T_i = 3092 \text{ K}$, $T_{iw} = 300 \text{ K}$ (m is the piston mass, and σ is the ratio between the masses of the working gas and piston). The initial kinetic energy of turbulence E_i was determined by the relationship $E_i = (3/2) \text{Tu}^2 \text{c}_0^2$, $\text{Tu} = 10^{-4}$ (Tu is the intensity of turbulent fluctuations). The scale of turbulence at the initial instant is $L_i = 0.01R = 3.95 \times 10^{-5} \text{ m}$.

It is expedient to integrate (11)-(14), (18) and (19) numerically by using implicit finite-difference schemes. In this case as the mesh resolution increases along the transverse coordinate, it is not necessary to follow the constraints on the time step. Integration of (15)-(17) is performed on the basis of explicit conservative schemes [10]. The constraints on the relationships between the time spacings and the longitudinal coordinate that occur here are associated with the velocity of perturbation propagation over the stream (and also with the rate of change of the process characteristics in time) and correspond to the substance of the matter. For relatively low Reynolds numbers (Re $\leq 10^5$) the integration of (18) and (19) is executed in the ordinary way. An increase in Re requires the introduction of a near-wall function for the scale of turbulence. In this case (18) operates in a domain standing a distance δR from the wall. The value of δR is determined by the condition y^+ = $u_{\star}(R-r)/v = const$ ($u_{\star} = \sqrt{\tau w/\rho}$ is the dynamic velocity). The magnitude of the constant was selected analogously to [11, 12]: $y^+ = 30$, which corresponds to the buffer layer domain. Introduction of the near-wall function for L with a linear distribution of the scale of turbulence over the transverse coordinate in the viscous sublayer and in the buffer zone assures universality of the profile of L in a broad Re range for incompressible fluid flow behind a hydrodynamic stabilization section.

Up to now only a quite limited quantity of investigations has been performed for the nonstationary heat elimination under Lagrange problem conditions. Among them is [13-15]. An attempt is made in [14, 15] for a possibly more complete description of the hydrodynamic and thermal processes in the domain behind the piston. A turbulence model based on the concept of the displacement pathlength is considered to compute the flow in the boundary layer domain. Let us note that all these papers are executed within the framework of sufficiently approximate formulations: not taken into account are the real law of piston motion, the finiteness of the volume of the space behind the piston, the influence of nonstationarity on the turbulent stream configuration, etc. It is difficult to conduct experiments under similar conditions because of the brevity of the process of propelling a body by a gas stream.



Experimental data on accelerated flows for high Re numbers are quite limited. The reliability of the computations represented in the paper were confirmed by experiments [16] where nonstationary incompressible fluid flow was investigated behind a hydrodynamic stabilization section with a linear law of variation of the mass flow rate G with time. Appropriate graphs of G(t) (dashed line) and integral characteristics of the dynamic action of the stream on the wall are represented in Fig. 1, where G_2 is the constant mass flow rate after emergence into the stationary flow mode with $\text{Re}_2 = 10^4$, $T_0 = \pi D^3/(4G_2)$, τ_{W2} is the shear stress for steady flow with mass flow rate G_2 , and the dash-dot curve displays the nature of distribution of the values of τ_W/τ_{W2} computed by the $E - \varepsilon$ model [17]. Good agreement with experiment is obtained when using the described model. The solid line corresponds to a = 0.0375. Numerical investigation of stationary turbulent incompressible fluid motions in a type behind a hydrodynamic stabilization section for $\text{Re} \in 10^5$, 10^7 shows that starting with $y^+ = 100$ and down to the channel axis the distributions of the relative values of the kinetic energy of turbulent fluctuations E/u_x^2 in Y = 1 - r/R in the domain are independent of Re.

Results of calculations are presented in Fig. 2: curve 1 is a computation performed under the conditions of an experiment [18] (stationary incompressible fluid flow, $Re = 4.235 \times$ 10^5), the points are test data, the dashed curve is Re = 5 × 10^6 , while 2 illustrates the turbulence kinetic energy profile obtained for a gas flow under Lagrange problem conditions at the time $t/t_0 = 2.5$ in the section $z = \xi/\xi_0 = 0.5$ ($t_0 = \ell_c/c_0$, ξ_0 is the total gas mass). As is seen from the graph the profile E/u_*^2 differs from the self-similar in an essential manner for nonstationary gas motion behind the acceleration piston. A more complete pattern of turbulence development in the domain behind the piston is represented in Figs. 3 and 4. Thus, the level lines for E/u_{x}^{2} corresponding to the value 0.01 are displayed in Fig. 3. The isolines 1-4 are constructed for $t/t_0 = 1.5$, 2.5, 3.01, 4.5. It is seen how the dimensions of the turbulized flow domain increase with the lapse of time. There are practically no fluctuations in the stream core. However, in the domain directly adjoining the piston a zone of elevated turbulent fluctuation intensity exists that occupies the whole transverse section whose dimensions also increase with the lapse in time. Isolines of different intensity at the times $t/t_0 = 4.5 (1-7 \text{ for } E/u_x^2 = 200, 100, 30, 8, 6, 4, 2)$ are represented in Fig. 4. As calculations show, the stream nonstationarity noticeably influences the distribution of the turbulence scale. Vortices of maximal dimensions are concentrated in a part of the channel adjoining the wall. The field of turbulent fluctuations is correlated sufficiently weakly on the axis. The magnitude of the scale in the near-axis domain grows with time while the profile L(r/R) is self-similar in the domain adjoining the piston where the turbulence intensity is high.

Let us examine certain results of solving the thermal part of the problem. Under the conditions being considered the heat transfer is characterized by high intensity and signifi



cant inhomogeneity in time and along the channel length. Consequently, necessary for its theoretical study is taking account of the flow nonstationarity, the spatial inhomogeneity of the stream, and variability of the thermophysical properties of the medium. Because of the complexity of the physical phenomenon and the absence of reliable experimental data on stationary heat elimination, a different kind of assumption permitting simplification of the physical and mathematical model of the phenomenon is used for estimating the influence of the heat loss (on the wall) on the gasdynamic pattern of the process and the computation of the channel wall temperature mode. Three assumptions of general nature can be extracted on whose basis the majority of approximate approaches and methods for studying the thermal processes in the conditions under consideration: the heat transfer quasistationarity, the absence of influence of the spatial inhomogeneity of the stream, and the variability of the thermophysical properties of the gas on the drag and heat elimination laws.

Recommendations on the computation of the friction and Nusselt number coefficients under gas motion conditions behind an accelerating piston are given in this paper on the basis of solving the problem within the framework of a sufficiently general formulation (11)-(26). It is established that criterial dependences of the form [2] can be used in determining the friction stress and heat flux in the flow domain not catching the domain directly adjoining the piston and the fixed left boundary. Certain data of a computation of the parameters of the thermal and dynamical action of the stream on the channel wall are presented in Figs. 5 and 6 where the points correspond to computations using the modified relations [2]

$$\zeta = 0.131 \operatorname{Re}^{-0.18}(\Theta/T_w)^{0.215}; \tag{27}$$

$$Nu = 0.0162 \text{Re}^{0.82} \text{Pr}^{0.4} (\Theta/T_w)^{0.215};$$
(28)

$$\operatorname{Re} = D\rho u/\mu(T), \quad \operatorname{Pr} = c_p \mu(\overline{T})/\lambda(\overline{T}), \quad \Theta = \overline{T} + r_w u^2/(2c_p), \quad r_w = \sqrt[3]{\operatorname{Pr}};$$
(29)

$$\mathbf{f}_w = (\zeta/8)\rho u^2, \, q_w = \operatorname{Nu}(\Theta - T_w)\lambda(T)/D.$$
(30)

Here ζ is the friction drag coefficient, Nu, Pr are the Nusselt and Prandtl numbers, Θ is the stream stagnation temperature, r_W is the coefficient of restoration, \overline{T} is the mean mass gas temperature, and T_W is the temerature of the channel inner surface.

The dependences of τ_W on the dimensionless time t/t_0 are shown in Fig. 5. The results of computations represented in the graph correspond to the section x = -0.05 m. At the initial time the coordinate x = 0 corresponds to the piston left boundary. The dashed line here and in Fig. 6 are computations for $\varphi = 1$. As is seen from the graphs, a computation on the basis of an E-L model is in agreement with the results of calculations by means of the relationships (27) and (30) that take account of the influence of the temperature factor Θ/T_W . Distributions of q_W as a function of t/t_0 are presented in Fig. 6. Here there is also good agreement of the results of computing q_W by the scheme described above and by the relationships (28) and (30).

In conclusion, we note the following. Despite the fact that the nonstationarity exerts noticeable influence on the field of stream fluctuation characteristics, the hydrodynamic and thermal action on the wall is the same as in quasistationary flow.

As was noted above, turbulence is generated in the near-axis domain (with the exception of the zone directly adjoining the piston). Nevertheless, there is a turbulized flow domain in the near-wall part (50-60% as compared with the self-similar motion case). These effects of the change in the turbulent stream configuration cancel each other in the sense of determining the integral action on the channel wall.

Therefore, the computation of the friction stress and the heat flux at the wall can be performed by using quasistationary dependences (27)-(30) that take account of the influence of a temperature factor in an essential manner.

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